LOGIC EXERCISES: WEEK 3

QUESTION 1

Formalise the following argument (from week 1, question 1) in L_1, and determine whether or not it is propositionally valid.

> If the hog-sprocket is defective, the bendix is not going to function. And if the grunge pedal is defective, the thermoblaster is not going to function. If the cramp-iron is not working, it must be because the hog-sprocket is defective. And if the colour detector is not working, it must be because the grunge pedal is defective. The cramp-iron and the colour detector are not both working. So either the bendix or the thermoblaster is not going to function.

QUESTION 2

Use partial truth-tables to test the correctness of the following sequents. Give counterexample structures where appropriate.

    i. $(P \rightarrow (Q \vee R))$, $(Q \rightarrow R) \vDash (P \rightarrow R)$

    ii. $((P \vee Q) \rightarrow (R \wedge S)) \vDash (P \rightarrow R)$

    iii. $((P \vee Q) \rightarrow (R \wedge S))$, $(R \rightarrow T) \wedge \neg T) \vDash P$

    iv. $(((P \wedge Q) \rightarrow R) \wedge ((P \wedge \neg Q) \rightarrow \neg R)) \vDash (P \rightarrow ((Q \wedge R) \vee (\neg Q \wedge \neg R))$

    v. $((P \rightarrow Q) \wedge (R \rightarrow S))$, $(P \vee R)$, $((P \rightarrow \neg S) \wedge (R \rightarrow \neg Q)) \vDash (S \leftrightarrow \neg Q)$

QUESTION 3

Using the dictionary given, formalize the following English claims in L_2.

> Domain: { Albert, Beth, Cecil, Di }   a: Albert
>
> S^1: … smokes   b: Beth
>
> D^1: … drinks   c: Cecil
>
> L^2: … likes …   d: Di

    i.   Albert smokes, but Beth does not.

    ii.   All the smokers drink as well.

    iii.   None of the drinkers smokes.

    iv.   Only the drinkers smoke.

    v.   None of the drinkers likes Beth.

    vi.   Beth likes all the smokers.

    vii.   Beth and Di like Albert, but they don't like Cecil.

   viii.   Cecil likes anyone who drinks and smokes.

    ix.   Cecil likes anyone who drinks and anyone who smokes.

    x.   If everyone smokes, then everyone drinks.

    xi.   Everyone who smokes drinks as well.

xii.   There are no smokers who drink.

xiii.   There are no drinkers who smoke.

xiv.   Everyone is either a drinker or a smoker.

xv.   Both Beth and Cecil like all the smokers.

QUESTION 4

And now the other way round: still using the same dictionary, render the following into idiomatic an unambiguous English.

i. $\forall x(Sx \rightarrow \neg Dx)$

ii. $(Lab \land \neg Lba)$

iii. $\exists x(Sx \land \neg Dx)$

iv. $\exists x Lxb$

v. $\exists x Lbx$

vi. $(\forall x Lxx \rightarrow \exists x Lxc)$

vii. $\forall x(Lxx \rightarrow Lxb)$

viii. $\exists x(Sx \land Lxa)$

ix. $(\forall x \neg Sx \rightarrow \forall x \neg Dx)$

x. $\forall x(\neg Sx \rightarrow \neg Dx)$